

# On the macroscopic limit of nuclear collective motion and its relation to chaotic behavior

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In this note we concentrate on slow collective motion of isoscalar type at small but finite excitations, as given in nuclear fission, for instance. At the end we are going to examine how chaotic behavior might influence typical transport properties of nuclear dynamics. The essential features may be examined by introducing (a) collective variable(s). Typically, the time scale of such a  $Q$  is longer by about a factor of 5 to 10 than the one for the dynamics of the residual, "nucleonic" degrees of freedom. Then it is fair to assume that the latter are in a local equilibrium, the properties of which vary with  $Q$ . A tractable but realistic description of nucleonic motion may be based on the deformed shell model, which, in combination to the Strutinsky procedure, is known to allow one calculating the static energy as function of  $Q$  and  $T$  to satisfactory precision.

**A locally harmonic approximation:** For the model just explained the Hamiltonian  $\hat{H}(\hat{x}_i, \hat{p}_i, Q)$  will depend on  $Q$  in parametric way. To understand the dynamics in the neighborhood of some  $Q = Q_0$  one may expand to second order (thus establishing a *locally harmonic approximation*)

$$\hat{H}(Q(t)) \simeq \hat{H}(Q_0) + (Q(t) - Q_0)\hat{F} + \frac{1}{2}(Q(t) - Q_0)^2 \left\langle \frac{\partial^2 \hat{H}}{\partial Q^2}(Q_0) \right\rangle_{Q_0, T_0}^{\text{qs}}. \quad (1)$$

The term of second order is treated on quasi-static average specified by the  $\hat{H}(Q_0)$ . The coupling between the collective and the nucleonic degrees of

<sup>1</sup> Supported in part by the Deutsche Forschungsgemeinschaft

<sup>2</sup> I will be reporting mainly on topics developed together with F.A. Ivanyuk, D. Kiderlen, A.G. Magner and S. Yamaji, whose collaboration is gratefully acknowledged, but the whole responsibility for this text is on my side. If not stated otherwise, details may be found in my article in Phys. Rep. 284 (4&5) (1997) 137-380, for shorter reviews see also nucl-th/9703056 and /9809017.

<sup>3</sup> Prepared for the "V. Workshop on non-equilibrium at short time scales", Rostock, April 1998

freedom is given by the term in the middle. Its form allows for an application of *linear response theory* to get for the average induced force  $\delta\langle\hat{F}\rangle_t = -\int_{-\infty}^{\infty} ds \tilde{\chi}(s)(Q(s) - Q_0)$ .

**Local self-consistency:** To probe the behavior of *all* degrees of freedom one may introduce a time dependent "external field"  $f_{\text{ext}}(t)$  by adding  $f_{\text{ext}}(t)\hat{F}$  to the Hamiltonian (1). For the "collective" response function  $\chi_{\text{coll}}(\omega)$  one gets

$$\chi_{\text{coll}}(\omega) = \frac{\chi(\omega)}{1 + k\chi(\omega)} \quad \text{with} \quad \delta\langle\hat{F}\rangle_{\omega} = -\chi_{\text{coll}}(\omega)f_{\text{ext}}(\omega) \quad (2)$$

Within an "adiabatic picture", the coupling constant  $k$  is given by

$$-k^{-1} = \left\langle \frac{\partial^2 \hat{H}}{\partial Q^2}(Q_0) \right\rangle_{Q_0, T_0}^{\text{qs}} + (\chi(0) - \chi^{\text{ad}}) = \left. \frac{\partial^2 E(Q, S_0)}{\partial Q^2} \right|_{Q_0} + \chi(0) \quad (3)$$

with  $\chi^{\text{ad}}$  being the adiabatic susceptibility and  $E(Q, S_0)$  the internal energy at given entropy  $S$ . Different both to the case of  $T = 0$  as well as to the "diabatic picture" — corresponding to common RPA — the  $k$  depends sizably on  $T$ .

**Microscopic origin of irreversibility:** Already at quite small excitations a nucleus must be considered an open system, as it may decay by emission of  $\gamma$ 's, nucleons etc. Thus each level has a natural finite width. For any model state, this width is enlarged considerably by residual interactions  $\hat{V}_{\text{res}}^{(2)}$  which couple the "simple" states to "more complicated" ones. Because of the large increase of the density of levels this mechanism gets effective already at low temperatures. On the level of single particle motion the effects of the  $\hat{V}_{\text{res}}^{(2)}(\hat{x}_i, \hat{p}_i)$  may be parameterized through a complex self-energy  $\Sigma(\omega \pm i\epsilon, T) = \Sigma'(\omega, T) \mp (i/2)\Gamma(\omega, T)$ . The intrinsic response function  $\chi(\omega)$  is calculated after replacing the single particle strength  $\varrho_k(\omega) = 2\pi \delta(\hbar\omega - e_k)$  by  $\varrho_k = \Gamma(\omega) / ((\hbar\omega - e_k - \Sigma'(\omega))^2 + (\Gamma(\omega)/2)^2)$  with

$$\Gamma = \frac{1}{\Gamma_0} \frac{(\hbar\omega - \mu)^2 + \pi^2 T^2}{1 + [(\hbar\omega - \mu)^2 + \pi^2 T^2] / c^2} \quad (4)$$

and  $\mu$  being the chemical potential. The  $\Gamma$  may be said to represent "collisional damping". In numerical computations, the following values have mostly been used for the parameters entering here:  $\Gamma_0 = 33$  MeV and  $c = 20$  MeV. The width  $\Gamma(\omega, T)$  takes on sizable values already at moderate excitations  $\hbar\omega - \mu$  above the Fermi surface  $\mu$ . For such reasons mean field approximations do not appear adequate at finite thermal excitations.

**Damped collective modes:** The microscopic damping mechanism forces collective motion to be damped, as may be inferred from the collective strength

distribution given by  $\chi''_{\text{coll}}(\omega)$ . Take some individual peak of the latter located around  $\omega_1$  to represent a collective mode. This peak may be approximated by the response function of a damped oscillator according to  $[\chi_{\text{coll}}(\omega)]^{-1} \simeq [\chi_{\text{osc}}(\omega)]^{-1} = -\omega^2 M(\omega_1) - i\omega\gamma(\omega_1) + C(\omega_1)$ , defined by the transport coefficients of average motion: inertia  $M$ , friction  $\gamma$  and local stiffness  $C$ . The transfer of collective energy into "heat" of the nucleonic degrees of freedom is determined by (with  $q = Q - Q_0$ )

$$-\frac{d}{dt}E_{\text{coll}} \equiv -\frac{d}{dt} \left( \frac{M(\omega_1)}{2} \dot{q}^2 + \frac{C(\omega_1)}{2} q^2 \right) = \gamma(\omega_1) \dot{q}^2 \equiv T \frac{d}{dt} S \quad (5)$$

Often the friction coefficient may well be approximated by the following form of the so called "zero frequency limit"

$$\gamma \approx \gamma(0) = \int_{-\infty}^{\infty} ds \tilde{\chi}(s)s = -i \left( \frac{\partial \chi(\omega)}{\partial \omega} \right)_{\omega=0} \quad (6)$$

**The macroscopic limit:** Many models describe the nucleus as a macroscopic system, similar to a drop of nuclear liquid. Such a picture may be expected to represent a nucleus realistically at larger  $T$  where shell effects become less effective — as it is well known from studies of the static energy. With respect to the dependence on  $T$  our theory leads to the following behavior:

- Above  $T \approx 1.5 \dots 2$  MeV the inertia  $M$  drops to values close to those of irrotational flow. This is largely a consequence of the  $T$ -dependent coupling constant (3) in the "adiabatic picture", which implies the strength distribution to concentrate in a strongly damped mode at very low frequencies.
- At larger  $T$  friction  $\gamma$  reaches values in the range of the "wall formula"  $\gamma_{w.f.}$ , provided that we use the single particle width as given by (4).
- Neglecting in the  $\Gamma$  of (4) both the  $\omega$ -dependence as well as the "cut off"  $c$ , the  $\gamma$  would drop like  $T^{-2}$  for larger  $T$ , in *this sense* exhibiting features of two-body viscosity as the dynamics becomes "dominated by collisions".

To some extent such features are found also in a model in which a collision term is added to the Landau-Vlasov equation, but where (following the "Kiev school") the latter is solved for appropriate boundary conditions specified by the collective mode; see a forthcoming paper with A.G. Magner.

**The interplay of one- and two-body viscosity:** The behavior at large  $T$  should not disguise the fact that for the nucleus, as a small and self-bound Fermi system, the mean field cannot be discarded totally. On the other hand, and as indicated earlier, to us it does not make much sense either, to neglect the coupling to more complicated states. One may ask the question whether or not it may be possible to disentangle the two components of friction. Inspect-

ing the microscopic formulas such a conjecture does not appear meaningful. The main reason is found in the fact that the effects of the mean field, as visualized through the one body operator  $\hat{F}$ , and of the residual interaction are interwoven in a highly non-linear way, as may be read off directly from the structure of the collective response function (2). At very small excitations, on the other hand, say at  $T = 0$ , nuclear friction is bound to become very small if not exactly zero. This is due to the presence of a gap in the low energetic spectrum of nucleonic excitations. For an inclusion of pair correlations into the present formulation see a forthcoming paper with F.A. Ivanyuk.

**The wall formula:** It has been demonstrated in various ways that and how the  $\gamma(0)$  of (6) turns into the  $\gamma_{w.f.}$  of the "wall formula". Generally speaking, the latter is reached as a macroscopic limit of the  $\gamma(0)$  if evaluated within the picture of *pure independent particle motion*, i.e. for  $\hat{V}_{\text{res}}^{(2)} = 0$  or  $\Gamma = 0$ . This limit may be defined literally by letting the size of the system become infinitely large or by applying Strutinsky smoothing procedures. In the former case collective dissipation may indeed arise from collisions of the particles with the moving "wall", with irreversibility showing up in a kind of "thermodynamic limit". Recently, a Strutinsky smoothing has been applied to the model where a finite system of nuclear dimensions, but consisting of independent particles, is forced to undergo shape vibrations, following some oscillating external source. Treating the latter within time dependent perturbation theory, the excitation left in the system after one cycle of the external field shows the typical strength function behavior, as function of the vibrational frequency. Applying a Strutinsky smoothing to these strength functions it is seen that the averaged energy behaves like expected from wall friction; for details see nucl-th/9709043. It is important to note that typically such smoothing procedures involve averaging over intervals in energy of the order of 10 – 20 MeV. This fact allows one to understand physically why the wall formula does not reflect any shell effects, and, hence, that it is insensitive to changes in  $T$ .

**Heat pole:** The (dissipative part of the) response function is associated to the correlation function by the fluctuation dissipation theorem (FDT)  $\psi''(\omega) = \hbar \coth(\frac{\hbar\omega}{2T}) \chi''(\omega)$ . In case that the intrinsic states have zero width the  $\psi''(\omega)$  has the following structure  $\psi''(\omega) = \psi^0 2\pi\delta(\omega) + {}_R\psi''(\omega)$  with  $\psi^0 = T(\chi^T - \chi(0))$  and  $\chi^T$  being the isothermal susceptibility. In analogy to transport in infinite systems the contribution at  $\omega = 0$  may be called the "heat pole". Due to the damping mechanism mentioned above a smooth peak develops

$${}_0\psi''(\omega) = \psi^0 2\pi\delta(\omega) \quad \Longrightarrow \quad {}_0\psi''(\omega) = \psi^0 \frac{\hbar\Gamma_T}{(\hbar\omega)^2 + \Gamma_T^2/4} \quad (7)$$

Its width may estimated to be about twice the single particle width at the Fermi surface  $\Gamma_T \approx 2\Gamma(\mu, T)$  (which in turn is close to  $2T$ ). According to (6)

the heat pole contributes to friction by the amount  ${}_0\gamma(0) \equiv 2\hbar\psi^0/(T\Gamma_T)$ .

**Nuclear ergodicity:** The residue of the heat pole reflects ergodic properties, which may become apparent after introducing the adiabatic susceptibility:  $\psi^0/T = \chi^T - \chi(0) = (\chi^T - \chi^{\text{ad}}) - (\chi^{\text{ad}} - \chi(0))$ . For the nuclear case the difference  $\chi^T - \chi^{\text{ad}}$  can be shown to be small. The difference  $\chi^{\text{ad}} - \chi(0)$ , on the other hand, is known to vanish if two conditions are met: The system should have no degeneracies and the spread in the energy of the states contributing should be sufficiently narrow. The latter property may cause problems for the nuclear case if the canonical distribution must be applied. The first property definitely will do so for the model of independent particles, but *adequate inclusion of  $\hat{V}_{\text{res}}^{(2)}$  can be expected to supply sufficient level repulsion*. Indeed, an application of the *Random Matrix Model to the linear response approach* is seen to simulate ergodic behavior. However, it must be said that our present way of handling the  $\hat{V}_{\text{res}}^{(2)}$  through self-energies is not good enough as by way of the form (4) degeneracies are not lifted. This is unfortunate, as otherwise our formulation allows one to include the many important features of collective motion mentioned above, which as yet, and to the best of my knowledge, are not amenable by applications of the RMM, in particular not at smaller excitations.

**Traces of chaotic motion:** Nuclear dynamics is known to exhibit chaotic features. The most prominent one is that given by *Wigner's law for the distribution of levels of the compound nucleus*, as seen in neutron resonances, for instance, thus being associated to an excitation of the order of the neutron binding energy. This property is *intimately connected to the existence of the residual interactions  $\hat{V}_{\text{res}}^{(2)}$  mentioned before, which couple simple configurations to "more complicated ones"*. The simple ones may be understood of being those of the *bare shell model*, viz those of the mean field approximation, which would lead to the *Poisson distribution not seen in experiment* (in the range of excitations mentioned before). In addition, for a typical mean field the *motion of a nucleon itself may already show some chaotic behavior*. This can be expected to be the more so the more this field is deformed; but notice that this fact does not rule out the appearance of shell effects (and thus of regular behavior) at finite deformations. Certainly, the study of such phenomena in their own are of considerable interest. In *connection to transport theory* however, it may be vital to *understand in which way chaotic dynamics is related to dissipative behavior*. Without any doubt, such a relation exists, indeed, in as much as  $\hat{V}_{\text{res}}^{(2)}$  is involved; please recall the remarks made in the previous section. However, there are claims that the *"wall formula" can be justified on the basis of nucleonic motion within a sufficiently complex potential*, discarding any  $\hat{V}_{\text{res}}^{(2)}$ . In our understanding this statement is *not justified*, for two reasons: (i) At finite  $T$ , the  $\hat{V}_{\text{res}}^{(2)}$  *must not* be neglected, according to (4) its influence even increases with  $T$ . (ii) The very form of the wall formula — if applied to independent particle motion — does involve averaging over

large intervals which in themselves wipe out any other microscopic details, see again nucl-th/9709043. In addition it may be stated that in our microscopic computations hardly any traces have been seen that friction would become bigger for more complex shapes.